Further Pure Core – Method Of Differences

Patrons are reminded that if something looks like it can be split in partial fractions, then that is probably a good thing to do.

1. (a) Show that $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$.

   (b) Hence find an expression, in terms of n, for $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!}$.

2. (a) Find an expression, in terms of n for $\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}$.

   (b) Show that $\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$

3. (a) Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$.

   (b) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$.

   (c) Find $\sum_{r=4}^{\infty} \frac{2r+1}{r^2(r+1)^2}$.

4. (a) Find $\sum_{r=1}^{n} \frac{2r-4}{r(r+1)(r+4)}$. $\boxed{\frac{1}{12} + \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3} - \frac{1}{n+4}}$

   (b) Hence find $\sum_{r=8}^{\infty} \frac{2r-4}{r(r+1)(r+4)}$.

5. (a) Find $\sum_{r=1}^{n} \frac{8}{8r^3 + 36r^2 + 46r + 15}$. $\boxed{\frac{2}{15} - \frac{1}{2n+3} + \frac{1}{2n+5}}$

   (b) Hence find $\sum_{r=n}^{\infty} \frac{8}{8r^3 + 36r^2 + 46r + 15}$.

6. Find $\sum_{r=1}^{n} \frac{3r^2 + 9r + 4}{r^2 + 3r + 2}$. $\boxed{3n + 1 - \frac{2}{n+2}}$

7. Find $\sum_{r=1}^{n} \frac{2r^3 + 2r^2 + 3}{r^2 + r}$. $\boxed{3 + n(n+1) - \frac{3}{n+1}}$